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Investigation of Newtonian Fluid Flow through a Two-dimensional Sudden Expansion and Sudden Contraction Flow Passage

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Abstract*—The importance of flow through sudden change in flow passages lies in the numerous industrial applications such as, polymer processing, injection molding, biomedical instruments, extrusion, thermoforming etc. In this work, fluid flow through an axi-symmetric sudden contraction flow passages has been carried out using commercial CFD tools. Flow in the inverse direction, which represents flow through a sudden expansion, has also been simulated for the same conditions. A comparison has been established between the two flows occurring in the opposite direction. A two-dimensional computational domain has been chosen for the analysis considering steady, laminar flow and Newtonian fluid. The effects of flow rates are investigated with constant diameter ratios to understand the flow characteristics for sudden expansion and sudden contraction geometry. The size of the recirculation zone, flow reattachment length, redevelopment of flow, recirculating flow strength depends on several parameters, primarily on flow rate i.e. Reynolds number, expansion/contraction ratio and flow direction. Flow tends to become unstable at lower Reynolds number for high expansion ratio.

***Keywords*—Sudden Contraction, Sudden expansion, Recirculation, Flow Separation, Flow Reattachment.**

I. INTRODUCTION

The flow of a fluid through a channel having a section, where sudden contraction or sudden expansion of the geometry takes place is encountered in many engineering applications such as tubular heat exchangers, capillary-tube viscometry, polymer processing, biomedical instruments, thermoforming, various manufacturing processes, fiber spinning, extrusion, injection molding, biomedical instruments, flow of refrigerant, etc. Thus, the behavior of flow field in the presence of sudden expansion geometry has been regarded as one of the most fundamental phenomenon of study in the field of fluid mechanics. Also due to the relative simplicity and economical feasibility, it is possible to conduct extensive numerical and experimental studies on the subject. With the presence of various dissipative behaviors such as boundary layer separation, recirculation, flow redevelopment, etc. flow through abrupt change in flow geometry requires extensive research.

Although, the behavior of the various types of Newtonian and non-Newtonian fluids through the geometry of sudden contraction has been studied in many ways, till now contraction flow presents some unresolved issues from fundamental point of view. Hence the need is felt to investigate the contraction flow characteristics with the variation of different flow parameters. Astarita and Greco [1] studied the excess pressure drop through contraction geometry with the help of experimental and analytical approaches. Their work contributes to the Hagenbach and Couette correction for the pressure loss correlations. Christiansen *et al.* [2] have reported the numerical work of flow through combinations of stream tube-real tube and real tube-real tube contractions. They presented the radial and axial-velocity profiles at the contraction region for various Reynolds number (*Re*). Numerical study of a fluid flowing into a circular tube from a reservoir has been reported by Vrentas and Duda [3]. Their work does not include the three general assumptions of zero excess viscous pressure dissipation, uniform velocity profile at inlet and the use of the parabolic, asymptotic form of the flow governing equations to describe the flow. They found the flow behaviors reach asymptotic nature at *Re*=200 and CR=4. The work suggests the absence of vena-contracta at these ranges of the parameters. Experimental and simultaneous numerical work has been presented in the study of Durst and Loy [4]. Using LDA measurement technique, they measured axial and cross-velocity components to study the velocity profiles at various axial locations as well as the size of the two separation zones at the convex and concave corners of the geometry. Their study also provides finite-difference computations using non-uniform grids. As their experiment was one of the earliest used optical methods in these fields, the authors encountered few measurement uncertainties and suggested corrective measures for future works.

Mitsoulis and Vlachpoulos [5] applied finite-element method for the solution of the Navier-Stokes equations in a sudden contraction geometry. Divergence of the residuals for the numerical model was found after a certain Re and radius ratio. Ozalp *et al.* [6] have performed flow visualization experiment of flow through a vertical sudden contraction with a fixed Contraction Ratio, CR=4 by using Particle Image Velocimetry (PIV) technique. They observed the stream-wise and span-wise velocity contours, for different Re, at the upstream of the contraction just before the abrupt change in geometry takes place. Chiang *et al.* [7] applied the primitive variable formulation over the Stream function- Vorticity approach. In their 3D contraction geometry, they found the existence of symmetry-breaking bifurcations corresponding to the downstream separation zones at very high Re. Some of the other relevant works on contraction flow deal with flow of non-Newtonian fluids [8-10] and multiphase fluids [11-15] have been reported. Also, work considering sudden expansion flow geometry both numerically and experimentally has been reported [16-19].

Although, the behavior of flow through a sudden change in flow passages has been studied extensively in the past few decades as to observe the separation and other phenomenon associated with it, the study of the previous works reveals that most of the works has been done in the turbulent region of the flow regime. The various aspects of flow in the laminar and the critical zone are still in need of more study. Also proper flow distribution in the sudden contraction or sudden expansion geometry is essential to achieve the desired flow characteristics. Hence the need is to investigate the sudden contraction and or sudden expansion flow characteristics with the variation of wide range of flow properties and geometric parameters. It is observed that the formation of separation eddy at the corner of upstream section of the conduit as well as the accelerated flow at the contraction zone are some of the characteristics of this type of flow. Also, for the same channel configuration, if the flow direction is reversed, it becomes a case of a flow through a sudden expansion. The study attempts to compare the flow situations in these two cases.

II. GOVERNING EQUATIONS AND NUMERICAL METHODS:

The geometric configuration of the two-dimensional flow passage is shown in Fig. 1. In case of contraction flow geometry, fluid flows axially through the inlet passage of width *D* and then passes through the sudden contraction zone of width *d*. The passage is vertically placed with its inlet at the top, thus the major driving force for the flow is gravity. The Contraction Ratio ($CR=\beta=D/d$) is defined as the ratio of the upstream passage width to downstream passage width. For the cases of expansion, where the flow direction is reversed through the passage, the Expansion ratio has been defined as above. The passage length for width *D* and *d* have been chosen as $l_D= 12D$ and $l_d= 3D$ respectively. The contraction ratio for the geometry varies as $\beta=D/d=2$, 4 and 8. Similarly, for the case of expansion flow, the expansion ratio values have been chosen as that of the *CR* values in the study of the contraction. Also, the channel with length l_D, works as the upstream channel for the contraction flow. It becomes the downstream channel for the expansion flow.

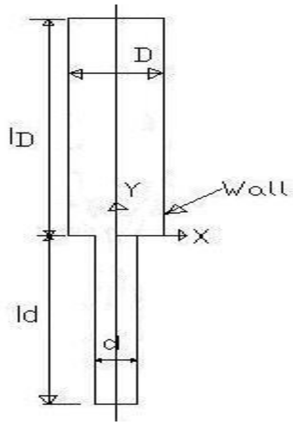


Fig. 1: **Computational Domain of Flow Geometry**

The boundary conditions considered for the numerical solution are uniform velocity at inlet, normal to the inlet boundary; due to the large length of the downstream passage, strong one-directional flow situation has been considered at outlet, thereby specifying the outflow boundary condition. No-slip boundary condition is taken on the walls. The flow is assumed to be steady, incompressible, Newtonian, isothermal, axisymmetric and laminar. The conservation form of governing equations for continuity and momentum for the steady flow field are given below.

$$\frac{\partial}{\partial x}(\rho U) + \frac{\partial}{\partial y}(\rho V) = 0 \qquad (1)$$

$$\frac{\partial}{\partial x}(\rho U^2) + \frac{\partial}{\partial x}(\rho UV) = -\frac{\partial p}{\partial x} + \mu(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}) \qquad (2)$$

$$\frac{\partial}{\partial x}(\rho UV) + \frac{\partial}{\partial x}(\rho V^2) = -\frac{\partial p}{\partial y} + \mu(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}) \qquad (3)$$

The numerical simulation has been carried out using commercial CFD tools. The basis of the numerical simulations follows the finite-volume scheme, with a steady implicit solver; whereas the Power Law model as described by Patankar [15] has been used for the discretization of the momentum, the standard scheme for pressure and the SIMPLE algorithm has been applied for the pressure-velocity coupling. The gradient calculation for the flow field has been carried out using the Green-Gauss Cell-based method. The simulations have been carried out using a convergence criterion of 10^{-7} for the continuity and *x* and *y*-momentum equations. As the major driving force considered in the flow-field is gravitational force, (using the gravity specifying tool of the commercial CFD software) thus gravity has been considered acting in the negative *y*–axis direction.

III. GRID INDEPENDENCE STUDY

The triangular mesh with uniform grid spacing of 0.8 and 81600 cells have been chosen for the numerical simulation. In order to study the effect of mesh size, grid independence study has been performed. Two different grids have been generated with 64% (for coarser mesh) and 130% (for finer mesh) of the original grid size.

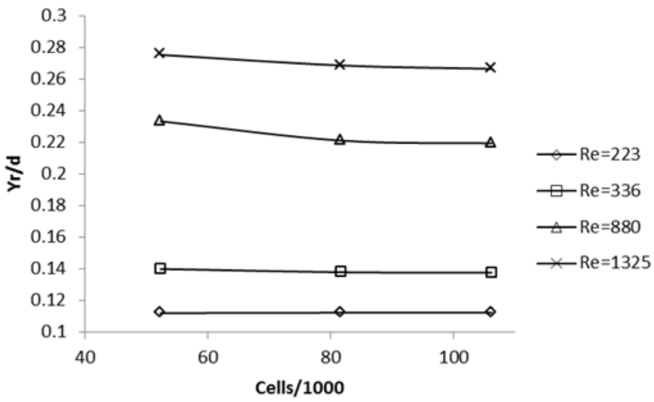
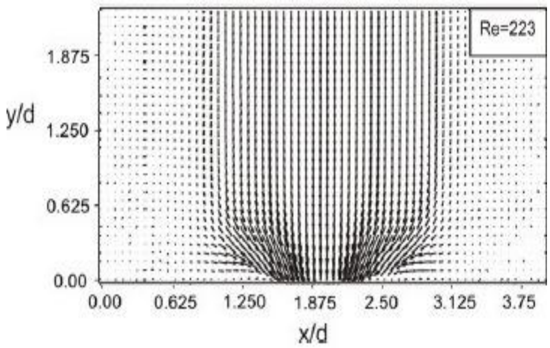


Fig. 2: **Recirculation Length at Corner of the Passage.**

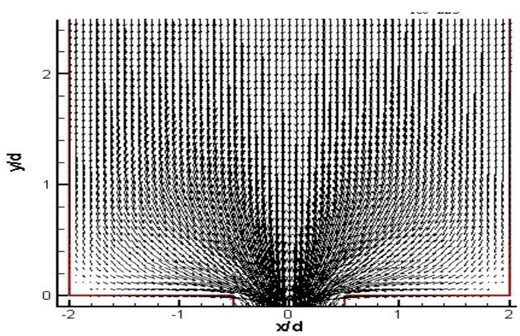
The grid with *52224* cells has been chosen for the coarse grid study of the problem, to get an initial idea about the numerical solution by using relatively small computational time. Due to the higher number of cells, time required to get the converged solution with the mesh with *106129* numbers of cells is much more than that required for the numbers of cells *81600.* one of the recirculation zone (before entry to the contraction zone) has been plotted against their respective grid size as shown in Fig. 2. The non-dimensional lengths of flow separation, y_r/d, are *0.112137* (at *Re=223*) for the coarser mesh with *52224* numbers of cells, *0.112368* for the chosen mesh with *81600* numbers of cells and *0.112438* for finer mesh with *106129* numbers of cells on the both wall. The result shows a good agreement for the three grids and very little variation has been observed as the grid size has been increased by almost *200%.* The mesh with *81600* numbers of cells has been chosen for further flow simulations. So the actual simulations are quite efficient, quantitative and consume less time to produce the numerical results.

IV. MODEL VALIDATION

The results of numerical simulations for the flow situation for the sudden contraction geometry, considering the present flow models, have been validated with the experimental results as reported by Ozalp and Pinarbasi, [6]. The time-averaged velocity vector plots on the *x-y* plane are presented for four different *Re* at the upstream of the contraction point are presented in Fig. 3, to observe the effect of the presence of the contraction zone on the flow field. Fig. 3(a-c) represents the time averaged velocity vector plots as reported by Ozalp [6] and Fig. 3(d-f) represents the time averaged velocity vector plots obtained with the present numerical simulations considering the same flow parameters. From the velocity vector plots (Fig. 3d-3f), it is observed that velocity vectors are parallel at the inlet of the channel and the flow is symmetric with respect to the vertical axis of the channel geometry.



(a)



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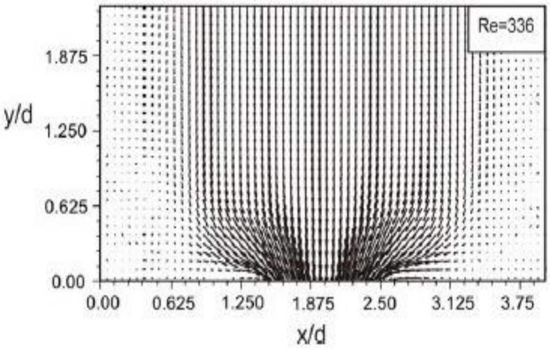
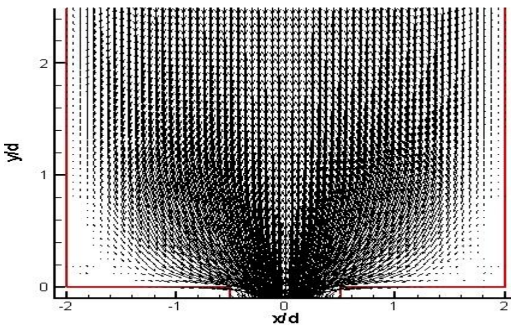
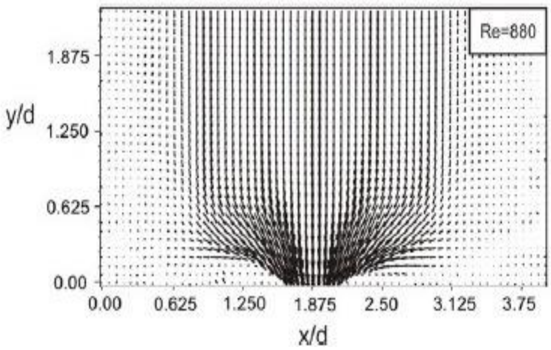
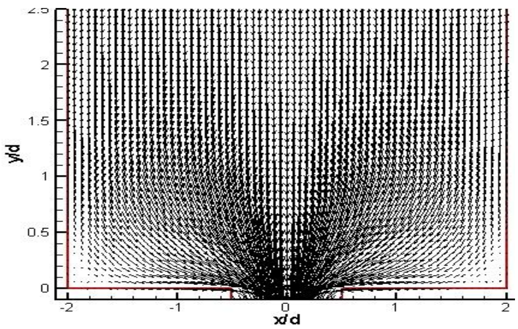




(b) **(e)**

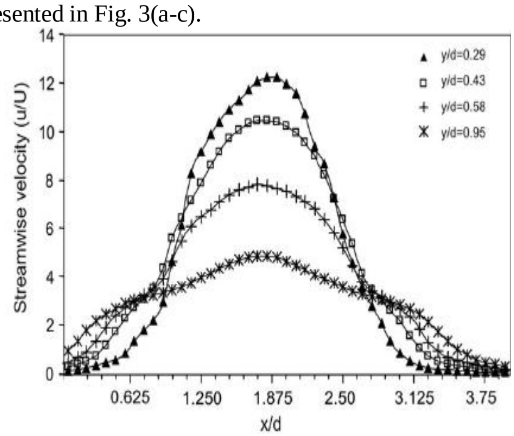




(c) **(f)**

Fig. 3 : **Time-averaged Velocity Vector Plots, (a-c) Experimental Results [*6*] and (d-f) Present Numerical Results.**

Near the contraction zone entrance, the flow velocity increases, as the flow area decreases and this is indicated by the dense vector distribution. Also the radial component of velocity increases in magnitude near the contraction zone. With the increase in Re, the flow velocity increases. The low vector density near the bottom corners of the figure indicates the formation of flow separation zone and the separated flow revolves at the corner. Though uniform velocity of flow was specified at the inlet, from the vector plot it can be concluded that near the contraction zone, the flow velocity is maximum at the center of the geometry, which suggests that the flow has developed from an initial parallel form to a parabolic form. From the above study, it is thus observed that the nature of the velocity vector plots and flow characteristics as presented in Fig. 3(d-f) as obtained with the present simulations, matches with the experimental results of Ozalp and Pinarbasi, [6] as presented in Fig. 3(a-c).



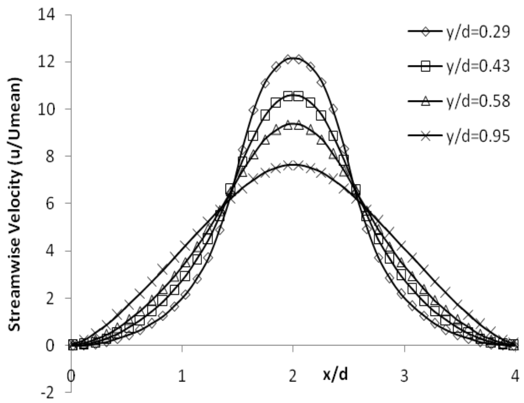
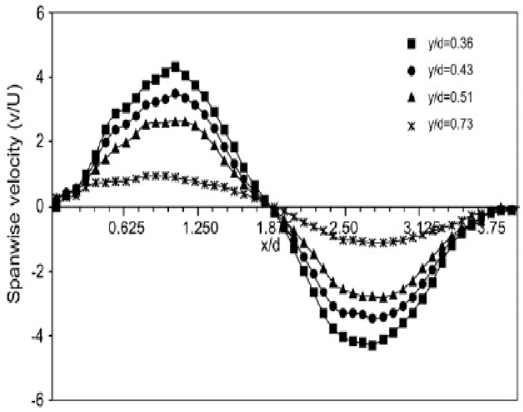


Fig. 4(a): **Stream-wise Velocity Plot at Different Axial Location Upstream of Contraction for *Re=223* [6].**
Fig. 4(b): **Stream-wise Velocity Plot at Different Axial Location Upstream of Contraction for *Re=223*, [*Present Study*].**

The axial and radial velocity profiles at different upstream locations (*y/d*) near the line of contraction have been presented in Fig. 4 and 5 or both the experimental work as reported by Ozalp and Pinarbasi, [6] and the present study. Fig. 4 represents the stream-wise flow velocity, *u* (Axial Velocity) distribution at different axial location (*y/d*) upstream of Contraction zone for *Re=223* as considered by Ozalp and Pinarbasi, [6]. The spatial co-ordinates have been modified to plot the distribution, the *x=0* point has been placed at the left wall, instead of the center as used in the numerical model. It is evident from the velocity distribution that the flow is axi-symmetric as the velocity reaches zero at the axis at both ends of the geometry. The results of the present study are very close to the experimental results for lower values of *y/d*. The velocity is the maximum at the center and this value increases as the probe is moved towards the contraction zone. This is also suggested from the velocity vector plot which indicates a higher concentration of velocity vectors near the contraction zone.

Though uniform velocity of flow was considered at the inlet, from the axial velocity distribution it can be concluded that the flow has developed from an initial parallel form to a parabolic form near the contraction zone.

The cross-stream wise flow velocity, *v* (Radial Velocity) distribution has been presented in Fig. 5 for *Re*=336 at four different upstream locations (*y/d*). The radial velocity is zero at the centerline and maximum at two locations at *x/d=1.5* and *2.5*. Similar to the distribution of the stream-wise velocity, here also the velocity increase with the proximity of the orifice but this is evident only between *x/d=1.0* and *3.0* for same *Re.*



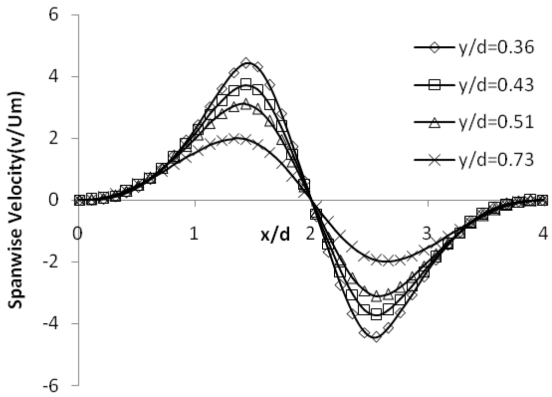
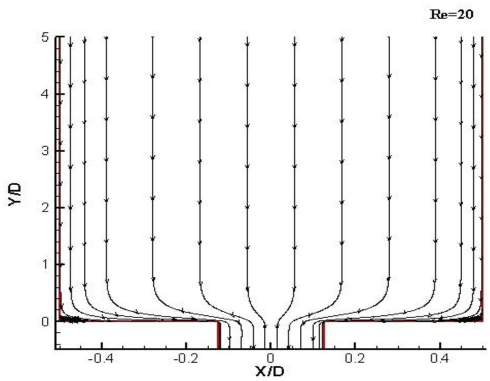


Fig. 5(a) : **Span-wise Velocity plot at Different axial location upstream of contraction for *Re=336* [6].*Fig. 5(b) : Span-wise Velocity plot at Different Axial Location Upstream of Contraction for *Re=336*, [*Present Study*].**

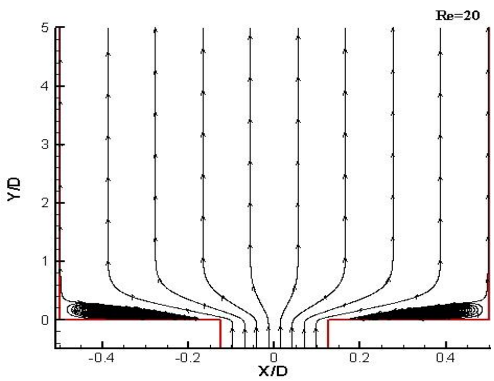
Axial velocity distribution, Fig. 4(a) and Fig. 4(b) shows excellent qualitative and quantitative agreement between the experimental results and present numerical results. Similarly, radial velocity distribution, Fig. 5(a) and Fig. 5(b) shows good agreement between the experimental results and present numerical results. Also the location of bubble separation, reattachment point and the flow vortex centre are same in both experimental and numerical result. Hence, these results represent the good agreement of present numerical simulations with the experimental results as reported by Ozalp and Pinarbasi, (*Ref.* [6]). Thus the above study (considering contraction flow geometry) represents the experimental validation of the present numerical results.

V. RESULTS AND DISCUSSIONS

In the present study, the point of interest is to observe the occurrence and the nature of various dissipative eddies at the convex and concave corners of the sudden change in flow geometry. The numerical analysis has been performed by considering geometrical configuration with $\beta=2$, 4 and 8, for two different flow directions for the same geometry. Thus one of the cases of flow, in the direction of gravity, will correspond to the contraction flow geometry. The other case, in which the flow takes place in the opposite direction of gravity, is the case of the flow through expansion flow geometry. The flow Reynolds number has been defined as $\mathrm{Re} = \frac{\rho U_{mean} D}{\mu}$, where U_{mean} is the mean inlet velocity while D is the width of the wider channel. The results of numerical simulations for the flow pattern in contraction geometry and for expansion geometry are presented by using streamlines plot (Fig. 6) for the purpose of a qualitative comparison between the two.



(a)



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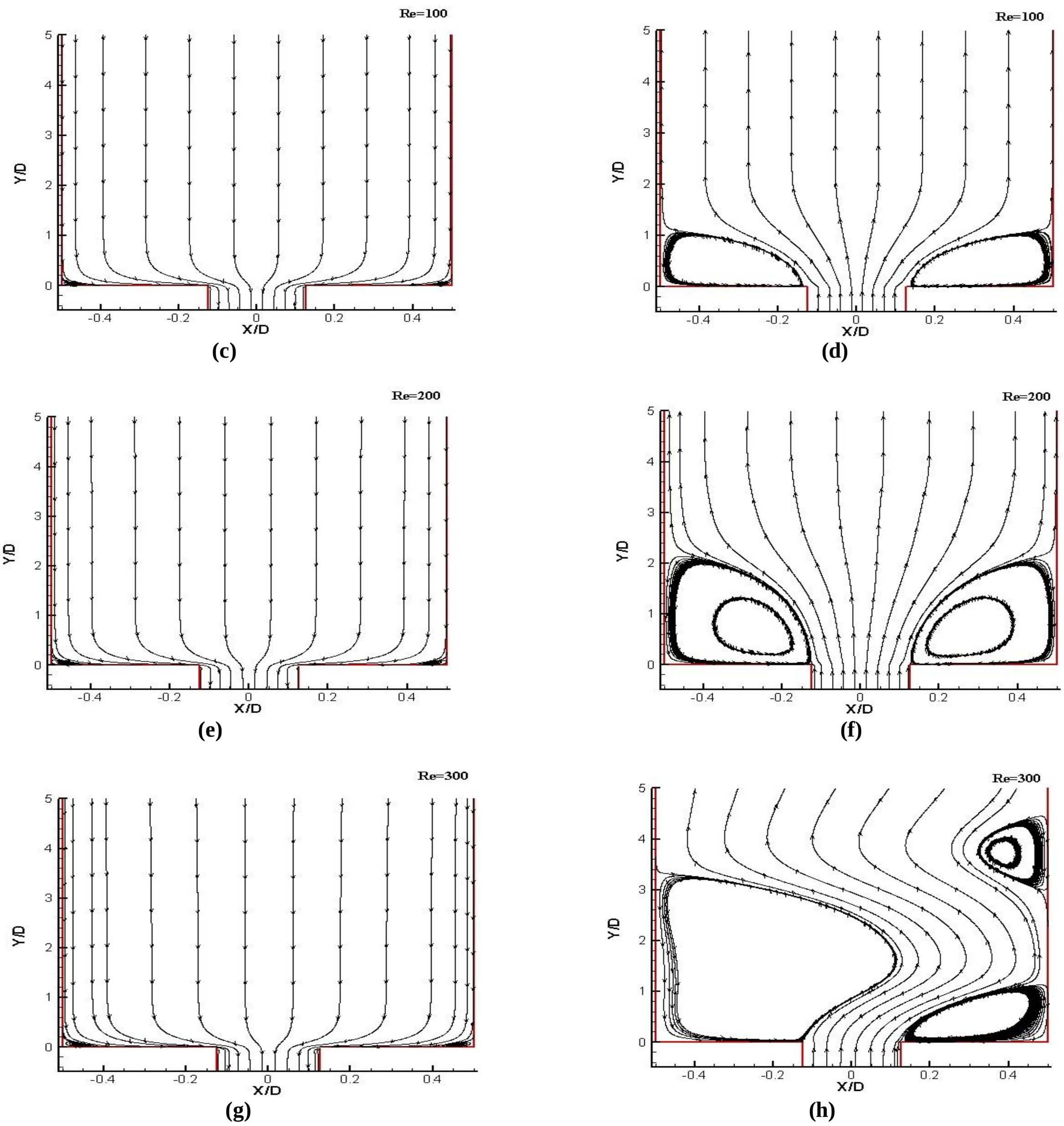


Fig. 6(a-h): **Streamlines Plot on the *x-y* plane for Contraction and Expansion section for β=4 with varying *Re*.**

From the streamlines plot, it is clearly observed that nature of the fluid flow is axi-symmetric with respect to the vertical axis of the channel, as the stream lines are parallel before (Fig. 6a) and after (Fig. 6b) sudden change in flow geometry at lower *Re*. From the velocity vector plot (Fig. 3d), it is observed that the axial velocity vector tends to concentrate more along the centerline region at the vicinity of the contraction zone. Thus the fluid flow seems to accelerate as the flow *Re* increases. Therefore, the flow separation and flow reattachment phenomena occurs at the concave wall at upstream of the contraction geometry (Fig.6a). Similarly, the flow separation and flow reattachment phenomena occurs at the concave wall at downstream of sudden expansion geometry (Fig.6b) for *Re*=20 and β=4. From stream line plot for sudden contraction geometry, it is also observed that there is negligible change in size of the flow vortex at the upstream of the contraction geometry with increase in *Re*; whereas with increase in *Re*, size and shape of the flow vortices changes at the downstream side in case of sudden expansion flow geometry.

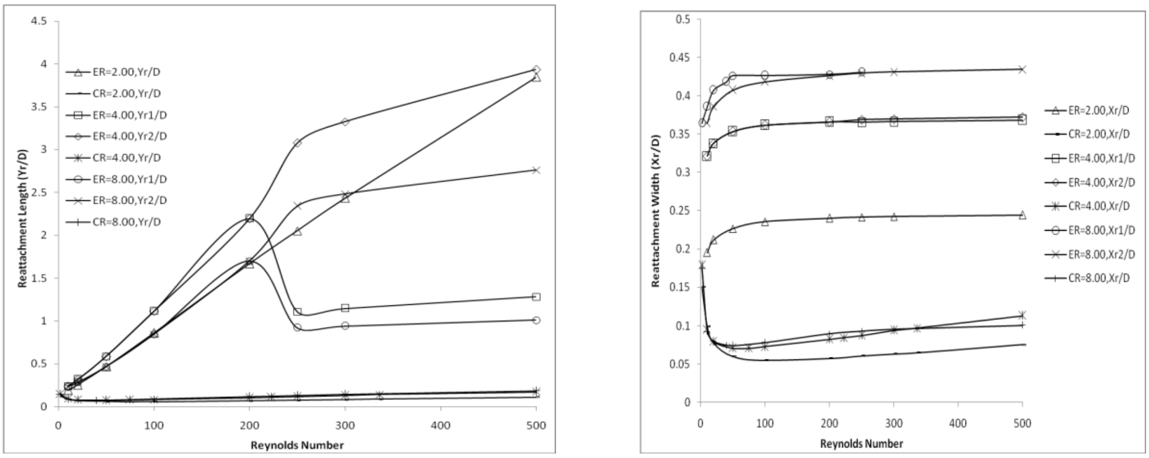
For the sudden expansion flow geometry, it is observed that flow reattachment length y_r/d (on both wall) increases towards the downstream side with increase in *ER* for same *Re*. But, there is no significant change in y_r/d incase of sudden contraction flow geometry. For *Re*≥200, β=4, flow asymmetry is clearly observed; as the size and length of the flow vortices (on both wall) are not same with respect to the vertical axis of the channel as shown in Fig. 6(h). This phenomena suggests that the flow has already entered the unstable regime, and also that the instability is more in the flow with higher expansion ratio (*ER*). For unstable flow field, the flow reattachment length y_{r1}/d at left wall differs with respect to the flow reattachment length y_{r2}/d at right wall and flow structure takes wavy shape. It is observed that at *ER*=4 and *Re*=300, the length of the flow reattachment point y_{r1}/d at left wall increases and at right wall y_{r2}/d decreases as shown in Fig. 6(h). Also there is a formation

of weaker secondary vortex (rotating in clockwise direction) at the right wall. The size of this secondary vortex increases with increase in *ER* for same *Re*. However, for sudden contraction flow geometry there is no flow asymmetry phenomena at the upstream of the flow passage.

Effect of length of the sudden change in flow geometry on the flow field also studied. It is observed that there is no influence of the channel length on the formation of annular separation bubble. Flow reattachment point, secondary or tertiary flow vortices. It indicates that the flow structure is independent of the length of the flow geometry for the present study.

Figure 7(a) represents plot for flow bifurcation point and flow reattachment length y_r/d (axial) at the down stream of the expansion zone and at the upstream of the contraction zone with the variation of *Re* for different *ER* and *CR*. The flow fields corresponding to the contraction geometry as well as the expansion geometry with ER=2.00 present an axi-symmetric formation of the separation regions on both walls. Thus only the length of one separation zone has been plotted. This plot also represents the good agreement between the present numerical study (considering sudden expansion flow geometry) and the numerical results as reported by Battaglia [18]. From the Fig. 7(a), it is observed that the flow reattachment length (both y_{r1}/d and y_{r2}/d) increases linearly with increase in *Re* for fixed *ER*. After a certain point y_{r1}/d increases and x_{r2}/d decreases sharply with increase in *Re* for a particular *ER*. This point is called Point of Bifurcation and corresponding Reynolds number is called Critical Reynolds number Re_{cr}. As the flow structure becomes unstable after Re_{cr}, flow symmetry about the vertical axis of the channel diminishes and the location of the center of the flow vortices (both at left wall and right wall) and also size of the vortices changes with change in *Re* beyond Re_{cr}. These phenomena changes for different *ER*. However, for sudden contraction flow geometry there is no existence of Re_{cr}, as the length y_r/d at the upstream of the contraction zone remains same for all *CR* and there is no significant change y_r/d with varying *Re*. For the flow in the downward direction (contraction flow geometry) separations form at the upstream corners but it is much smaller in size as compared to the reversed flow (expansion flow geometry), where the major separation occurs at the downstream. This can be attributed to the effect of gravity, increasing the separation for the upward flow.

Variation of radial length of flow separation zone both for sudden expansion and sudden contraction flow geometry, are presented in Fig. 7(b). From the plot it is observed that, a similar type of distribution was previously shown by Durst *et al.* [4]. This suggests the existence of a critical Reynolds number for the flow through sudden contraction. It can be seen that above this point the size of the recirculation zone increases almost linearly.



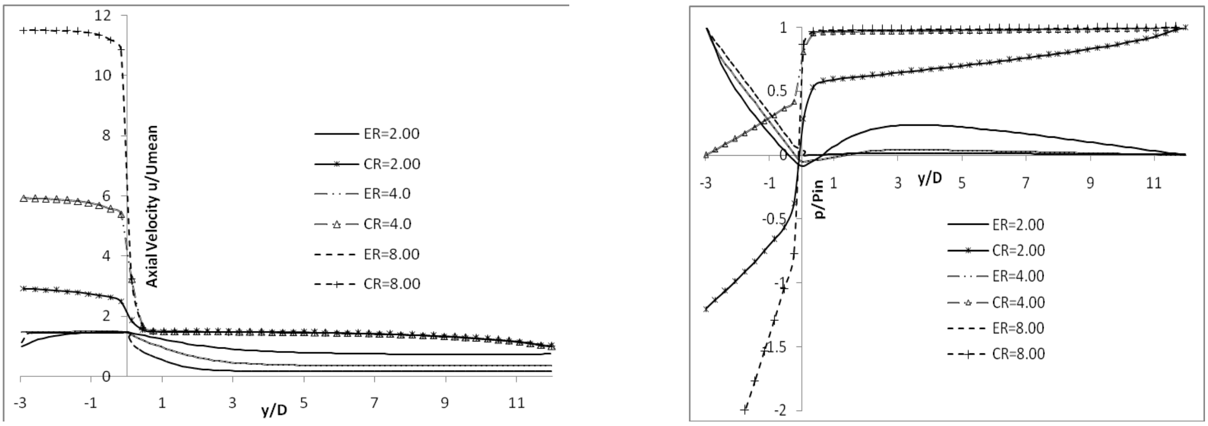
(a) ***(b)***

Fig. 7: **(a) Variation of axial Length (b) Variation of Radial length of flow Separation zone for Expansion and Contraction flow Geometry.**

However, for the case of the upward flow, which forms separation in the downstream corners of the channel, the critical point in the flow is the point where the flow at the downstream starts to become asymmetric. Although, it is seen that for the case of $\beta=2$, there is no asymmetry, for the cases of β=4 and 8, this instability is clearly visible above a particular value of *Re*. It can, therefore, be safely assumed that for the case of β=2 the critical Reynolds number exists but it is outside the range of the current study. The curve for β=2 shows very good linear variation of recirculation length and width with Reynolds number.

Figure 8(a) and (b) represents the variations of axial velocity and pressure along the centerline of the flow both for the sudden expansion and contraction flow geometry with three different size ratios (β) and at *Re*=223. From Figure 8(a), it is observed that, the dimensionless axial velocity (u/U_{mean}) increases with decrease in y_r/d and reaches maximum value at the point of sudden contraction point (y_r/d=0) for same *CR*. With the increases in *CR*, the peak value of u/U_{mean} also increases. Where as for sudden expansion flow geometry, u/U_{mean} decreases slightly after the point of sudden expansion with increase in y_r/d and there is smaller variation in u/U_{mean} with the increase in *ER* in comparison to the increase in *CR*. Similarly, from Figure 8(b), it is observed that, the dimensionless pressure (p/P_{in}) decreases gradually along the centerline with decrease in y_r/d and suddenly falls at the point of sudden contraction point (y_r/d=0) for same *CR*. Similar observation is found for varying *CR*. Whereas for sudden expansion flow geometry, p/P_{in} decreases with increase in y_r/d and reaches minimum value

at the point of sudden expansion and again p/P_{in} increases with increase in y_r/d and there is smaller variation in p/P_{in} with the increase in *ER* in comparison to the increase in *CR*. It is seen that the presence of the abrupt contraction is much more in the sudden variations of velocity and pressure, for the same inlet conditions, than that of a sudden expansion. There is an appreciable jump (rise) in axial velocity and drop in pressure for the former case. Whereas for the case of a sudden expansion, the variation is much more gradual and compared to the sudden contraction case, these variations are negligible. So it is observed that for the same channel size ratio, contraction affects the basic flow parameters drastically than the presence of a sudden expansion in the flow passage.



(a) *(b)*

***Fig 8 (a)* Variation of Axial Velocity along the Centerline for *Re*=223; (b) Variation of Pressure along the Centerline for *Re*=223.**

From the Fig. 7(a), it is clear that the bifurcation point (for sudden expansion flow geometry), that is the upper point of stability, shifts towards lower values of Reynolds number for flow geometries with higher expansion ratio (*ER*). This signifies the relative instability of these flow fields. The values of Re_{cr} for different *ER* are presented in Table 1. So from these analysis, it could be concluded that, for a fixed fluid flow rate, the flow tends to become unstable with the increase in expansion ratio or in other words the flow for higher expansion ratio becomes unstable at low flow rates and vice-versa for sudden expansion flow geometry. Whereas there is no such phenomena for sudden contraction flow geometry.

Table: 1

Expansion Ratio (*ER*)	**Critical Reynolds Number (Re_{cr})**
2	-
4	200
8	200

VI. CONCLUSIONS

In the present work, the analysis of several two-dimensional flow fields through sudden contraction and sudden expansion geometries under the action of gravity has been carried out numerically to get a better understanding of the sudden contraction and sudden expansion flow characteristics as well as to compare these two similar but opposing flow situations. The study has provided detailed numerical results for varying contraction ratio and expansion ratio (β) with varying *Re*. Physical characteristics of the flow of fluid for the case of the contraction flow, chosen from the study of Ozalp and Pinarbasi, [6], for the purpose of the experimental validation of present numerical results.

A good qualitative and quantitative agreement has been observed between the present numerical results (considering contraction flow geometry) and experimental results as reported by Durst F. and Loy T. [4]. From the streamlines plots, the presence of upstream and/or downstream separation zones is clearly observed for different size ratio (β) with varying *Re*. The sudden contraction, as compared to the sudden expansion, is seen to have drastic impact on the flow field for the same β. Gain of axial velocity (u/U_{mean}) along the centerline is maximum at the point of contraction with respect to the sudden expansion pint and the effect is more with hire *CR* ratio in comparison to the higher *ER*. For the flow in the downward direction (contraction flow geometry) separations form at the upstream corners but it is much smaller in size as compared to the reversed flow (expansion flow geometry), where the major separation occurs at the downstream. This can be attributed to the effect of gravity, increasing the separation for the upward flow. Due to the high viscosity of the fluid, at low Reynolds Number the two-dimensionality of the actual flow has been considered.

Nomenclature

β	Width Ratio = D/d
μ	Viscosity of Flowing Fluid, ($N\text{-}sec/m^2$)
ρ	Density of Flowing Fluid, (kg/m^3)
D	Width of the Inlet Flow Passage, (m)

d	Width of the Outlet Flow Passage, (m)
l_u	Length of the Passage Upstream of the Contraction, (m)
l_d	Length of the Passage Downstream of the Contraction, (m)
Re	Reynolds Number ($Re= \rho U_{mean}\, d /\mu$)
U_{mean} , U_m	The average Inlet Velocity, (m/sec)
x, y	Cartesian Coordinate System, (2D).
u	Axial Velocity, (m/sec)
v	Radial Velocity, (m/sec)
Y_r/D	Non-dimensional Recirculation Zone Height
X_r/D	Non-dimensional Recirculation Zone Width

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